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Experimental study of natural convection heat transfer of air in a cube below atmospheric pressure

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Abstract

Experimental study was carried out on the temperature profile of natural convection of air in a 50 mm cube with the temperature difference of 30 K between two opposing vertical walls. The objective is to investigate the essential aspect of heat transfer in the wide range of Rayleigh number (*Ra*) by lowering the pressure. The pressure was varied from 5.40 kPa (40.5 mmHg) to 99.99 kPa (750.0 mmHg). These correspond to $Ra = 1.04 \times 10^3$ to 3.56×10^5. The results show that the temperature distribution and Nusselt number approach those of conduction state as the pressure decreases.

Keywords: Natural convection; Low pressure; Pressure dependence

1. Introduction

In many problems of industrial engineering, the enhancement of heat transfer is crucial from the viewpoint of heat removal. However, the minimization of heat transfer is also important in the storage of energy, insulation of fluid and materials (e.g., transportation of high temperature fluid without heat loss), and so on [1–3].

Using the characteristic length, apparatus condition and fluid properties of a problem, heat transfer is related to the Rayleigh number (*Ra*) which is the product of the Prandtl number (*Pr*) and Grashof number (*Gr*). The effect of convection on heat transfer is expressed with the Nusselt number (*Nu*). Small *Ra* represents small amount of heat transfer, or repression of energy transfer. The weak convection results in *Nu* near unity.

There exist many studies on numerical calculation of natural convection [4–7]. Though numerical study on the heat transfer under small *Ra* is not difficult to perform, experiments with such conditions are difficult to pursue. This is due to the fact that the small *Ra* under the realistic experimental conditions requires small temperature difference or small characteristic length. For example, there are a number of experimental studies which deal with the heat transfer aspect in the wide range of *Ra* including the critical *Ra* for natural convection problem of fluid heated from below (Rayleigh–Bénard convection problem). However, the depth of fluid is required to be maintained of the order of 1 mm to attain the critical *Ra* [8,9].

Though an experiment on natural convection of a small *Ra* gives an essential aspect, this requires unrealistic characteristic length and temperature difference. Consequently, it is quite difficult to carry out such experiments, visualize flow pattern and measure some properties such as temperature. On the other hand, if a gaseous fluid is used, even a small *Ra* can be also achieved with realistic conditions by the reduction of pressure which is equivalent to decrease its density. However, there seems no study which investigates on the essential aspect of natural convection in an enclosure from the classical and basic point of view using dimensionless parameters. Accordingly, the basic experimental study on natural convection of a small *Ra* is considered

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Nomenclature

C_p	specific heat at constant pressure ($\mathrm{J\,kg^{-1}\,K^{-1}}$)
d	diameter of a molecule (m)
g	gravitational acceleration ($\mathrm{m\,s^{-2}}$)
Gr	Grashof number, $Gr = g\beta L^3 \Delta\theta / \nu^2$
k	Boltzmann constant ($\mathrm{J\,K^{-1}}$), $k = 1.38 \times 10^{-23}$ ($\mathrm{J\,K^{-1}}$)
Kn	Knudsen number, $Kn = \lambda / L$
L	characteristic length (m), one side length of a cube in the present study
Nu	Nusselt number, $Nu = -(\partial\Theta/\partial Z)_{\mathrm{conv}}$
p	pressure (Pa)
Pr	Prandtl number, $Pr = \nu/\alpha$
Ra	Rayleigh number, $Ra = PrGr$
x, y, z	Cartesian coordinate
X, Y, Z	dimensionless Cartesian coordinate, $(X, Y, Z) = (x, y, z)/L$
α	thermal diffusivity ($\mathrm{m^2\,s^{-1}}$), $\alpha = \kappa/(\rho C_p)$
β	thermal expansion coefficient ($\mathrm{K^{-1}}$)
θ	temperature (K)
$\Delta\theta$	temperature difference (K), $\Delta\theta = \theta_{\mathrm{h}} - \theta_{\mathrm{c}}$
θ_{av}	average temperature (K), $\theta_{\mathrm{av}} = (\theta_{\mathrm{h}} + \theta_{\mathrm{c}})/2$
Θ	normalized temperature, $\Theta = (\theta - \theta_{\mathrm{c}})/\Delta\theta$
κ	thermal conductivity ($\mathrm{J\,s^{-1}\,m^{-1}\,K^{-1}}$)
λ	molecular mean free path (m)
μ	viscosity ($\mathrm{kg\,m^{-1}\,s^{-1}}$)
ν	kinematic viscosity ($\mathrm{m^2\,s^{-1}}$), $\nu = \mu/\rho$
ρ	density ($\mathrm{kg\,m^{-3}}$)

Subscripts

air	air
av	average
c	cold wall
cond	conduction
conv	convection
h	hot wall

valuable to be investigated on. Especially, this kind of studies may serve in the fields encountered in semiconductor technology such as chemical vapor deposition process [10,11], and in the low pressure systems encountered in aerospace and vacuum engineering. As a gaseous medium, air is adopted in the present study.

The aim of this study is to investigate the essential aspect of natural convection heat transfer in a cube with the temperature difference imposed between two opposing vertical walls in the wide range of Ra by decreasing the pressure of a gas. The evidence of continuity of the used gas is also briefly illustrated, since this gives the reliability of the measured temperature in the present experiment. In this problem, convections occur for any Rayleigh number which is different from the Rayleigh–Bénard convection.

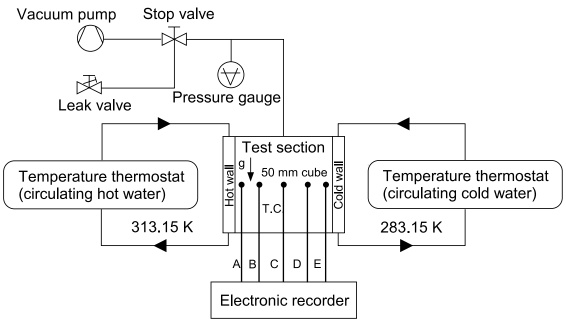

Fig. 1. Experimental apparatus system. The test section is a 50 mm cube constructed of four adiabatic transparent acrylic resin plates of 25 mm thick, and the hot and cold copper plates of 5 mm thick. These two plates are vertically opposing, and are maintained isothermally constant at 313.15 and 283.15 K by circulating hot and cold water, respectively.

2. Experimental facility and procedure

Fig. 1 shows the experimental system in the present study. The test section is a cube with effective one side length of 50 mm. This is constructed from two opposing vertical copper plates of 5 mm thick, and the other four insulated plates of a transparent acrylic resin whose thickness is 25 mm. The effective inside volume is equal to 1.25×10^5 $\mathrm{mm^3}$ and air is filled in this domain. Temperature difference is imposed through these two copper plates, and the temperatures of these plates were maintained isothermally constant with thermostats by circulating water and an electronic controller. The temperatures of hot and cold plates were set at $\theta_{\mathrm{h}} = 313.15$ K (40 °C) and $\theta_{\mathrm{c}} = 283.15$ K (10 °C), respectively. Accordingly, the temperature difference was equal to $\Delta\theta = 30$ K. Further, the averaged temperature $\theta_{\mathrm{av}} = (\theta_{\mathrm{h}} + \theta_{\mathrm{c}})/2$ was kept at 298.15 K (25 °C). The ambient temperature was also set at 298.15 K carefully to minimize heat loss from the test section. The inside pressure was measured by a diaphragm-type pressure sensor. In the present study, the pressure was changed from 5.40 kPa (40.5 mmHg) to 99.99 kPa (750.0 mmHg) by use of a vacuum pump.

All temperatures mentioned above were measured carefully with K-type (chromel–alumel) thermocouples of 0.18 mm diameter protected by sheath whose outer dia-

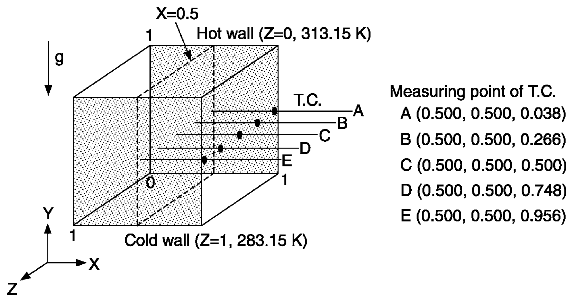

Fig. 2. Details of test section. The values in brackets are normalized coordinates (X, Y, Z) of measuring points of thermocouples. 1 corresponds to 50 mm in the dimensional system.

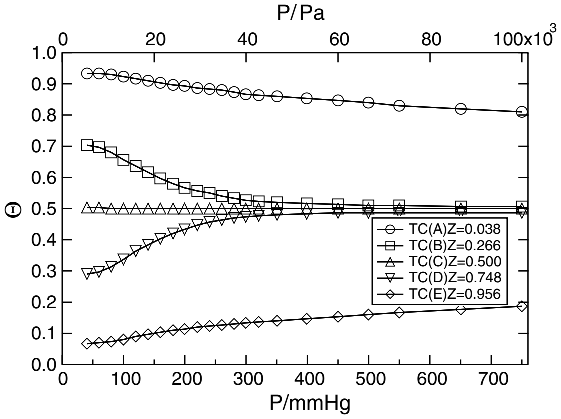

Fig. 3. Dependence of normalized temperature on pressure.

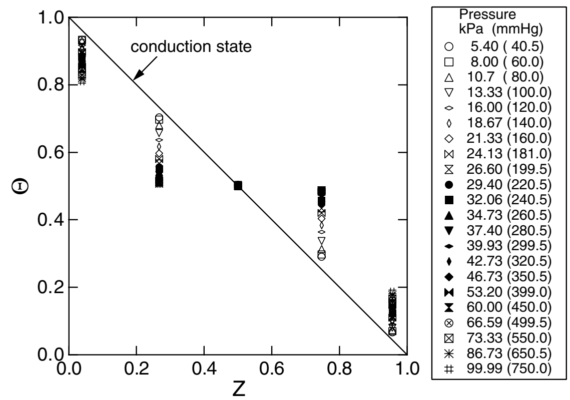

Fig. 4. Distribution of normalized temperature at each pressure.

meter is 1 mm, and these were calibrated by a primary standard thermometer in the present experimental range. The surface temperatures of copper plates were ascertained to distribute uniformly constant in another experiment by pasting the thermocouples on them with metal tape. The temperature was measured and recorded within ±0.05 K accuracy by an electronic recorder. The pressure sensor was calibrated by a reference sensor, and the measured pressure is within ±33 Pa (±0.25 mmHg) accuracy. The temperatures around the test section were measured and controlled with thermocouples and an electronic thermostat device. The experimental conditions were kept as steady as possible, and experimental repeatability can be assumed to be verified throughout the experiment.

Fig. 2 shows the detail of the test section. The arrangements of thermocouples are illustrated with its coordinate. Let the dimensional coordinates of a cube be (x, y, z), and the normalized coordinates as in Fig. 2 be (X, Y, Z) defined as

$$(X, Y, Z) = \frac{(x, y, z)}{L}. \tag{1}$$

Here, L is the one side length of the cube, and this is equal to 50 mm in the present study.

3. Results and discussion

3.1. Experimental data

Fig. 3 shows the dependence of temperature on the pressure of air. All thermocouples are constructed at $X = 0.5$ and $Y = 0.5$ as shown in Fig. 2. Here, Θ in Fig. 3 represents the normalized temperature and is defined as

$$\Theta = \frac{\theta - \theta_c}{\theta_h - \theta_c}. \tag{2}$$

Accordingly, the normalized temperature Θ varies from 0 at cold wall to 1 at hot wall. This figure illustrates that the temperature at each point apparently approaches some value as the pressure becomes small. Especially below 6.67 kPa (50.0 mmHg), the distribution seems to converge to some state. Θ at the center $(X, Y, Z) = (0.5, 0.5, 0.5)$ of the domain is found to be maintained almost 0.5 during the experiment.

Fig. 4 shows the temperature distribution at each pressure. The normalized temperature distribution appears to approach that of conduction state as pressure decreases. The plane at $Z = 0$ expresses the hot wall, and that at $Z = 1$ stands for the cold wall. This figure also shows that the temperature at the center of the domain is kept at $\Theta = 0.5$ (298.15 K). The almost linear distribution at low pressure changes due to the convection as the pressure becomes large.

3.2. Data reduction by Kn

Above experimental data on heat transfer in an enclosure can be reduced using the dimensionless number of Rayleigh number (Ra). Ra is defined as

$$Ra = Pr\,Gr, \quad Gr = \frac{g\beta(\theta_h - \theta_c)L^3}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}. \tag{3}$$

Here, g is the gravitational acceleration. α, β and ν are the thermal diffusivity, volumetric expansion coefficient and kinematic viscosity, respectively.

These dimensionless variables are valid for continuum medium, since these are defined with the properties of a bulk fluid. However, in the present study, air is used and its pressure is lowered. Accordingly, it is crucial if the employed air can be assumed continuous or not.

To investigate the degree of discontinuity, the Knudsen number (Kn) is generally used, and this is defined as follows from kinetic theory and equation of ideal gas [12],

$$Kn = \frac{\lambda}{L}, \quad \lambda = \frac{k}{\sqrt{2}\pi d^2}\frac{\theta}{p}. \tag{4}$$

Here, λ, k, d and p are mean free path of fluid molecules, the Boltzmann constant, diameter of a fluid molecule and pressure of gas, respectively. For nominally averaged air molecules of $d_{air} = 310$ pm [13], λ can be expressed as,

$$\lambda/\mathrm{m} = 3.23 \times 10^{-5}\ \frac{\theta/\mathrm{K}}{p/\mathrm{Pa}}. \tag{5}$$

Here, λ, θ and p are in the units of m, K and Pa, respectively. Considering the air at $\theta = 298.15$ K, the pressure p from 5.40 kPa (40.5 mmHg) to 99.99 kPa (750.0 mmHg) gives Kn of 3.57×10^{-5}–1.93×10^{-6}. From the viewpoint of kinetic theory, gas has been assumed continuous if Kn is less than about 0.01. Consequently, air at low density considering in the present study is ensured to be continuum.

3.3. Data reduction by Ra

Since the air adopted in the present study can be assumed continuous, Ra can be estimated if some properties of air at each pressure is obtained. In the present study, an estimation software [14] was adopted to calculate the properties included in Ra of Eq. (3) of air at low pressure. In the calculation using this, air is assumed to have the nominal composition of nitrogen (78.12 mol%), oxygen (20.96 mol%) and argon (0.92 mol%). Employing this, Ra which depends on the pressure can be predicted. Fig. 5 shows the dependence of Ra on the pressure using the properties of air confined in a 50 mm cube at $\theta = 298.15$ K in case of $\Delta\theta = 30$ K. Ra varies from about 1.04×10^3 to 3.56×10^5 corresponding to the pressure from 5.40 kPa (40.5 mmHg) to 99.99 kPa (750.0 mmHg). From kinetic theory [12], p is in inverse proportion to λ as in Eq. (4). Further, λ is proportional to both ν and α. Consequently, from Eq. (3), Ra is proportional to the square of p. Fig. 5 supports this nature. In the above mentioned estimation of

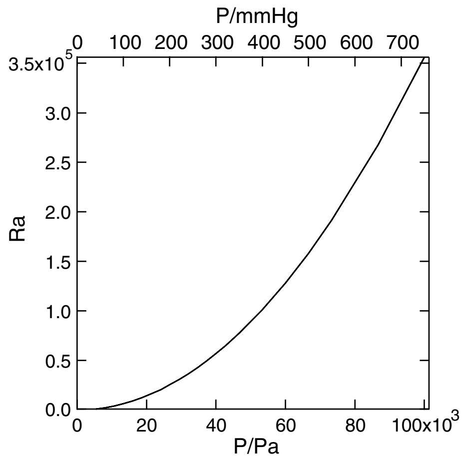

Fig. 5. Dependence of Ra of air on pressure below atmospheric pressure. The required air properties are calculated by an estimation software [14] at 298.15 K. Typical length and temperature difference are equal to 50 mm and 30 K, respectively.

Ra, μ, κ and β are almost independent of pressure, and only density increases with pressure. Accordingly, Pr is almost constant, and both α and ν decrease with pressure. For convenience, these are summarized as follows to predict Ra of air at 298.15 K below atmospheric pressure which is enclosed in a 50 mm cube imposed $\Delta\theta = 30$ K.

$$\begin{aligned} &\mu = 1.83 \times 10^{-5}\ \mathrm{Pa\,s}, \quad \kappa = 2.60 \times 10^{-2}\ \mathrm{W\,m^{-1}\,K^{-1}}, \\ &\beta = 3.35\text{–}3.36 \times 10^{-3}\ \mathrm{K^{-1}}, \\ &C_p = 1.00\text{–}1.01 \times 10^3\ \mathrm{J\,kg^{-1}\,K^{-1}}, \\ &\rho/(\mathrm{kg\,m^{-3}}) = 1.56 \times 10^{-3} \times (p/\mathrm{mmHg}) \\ &\qquad = 1.17 \times 10^{-5} \times (p/\mathrm{Pa}), \end{aligned} \tag{6}$$

$$\nu = \mu/\rho, \quad \alpha = \kappa/(\rho C_p), \quad Pr = 0.709,$$

$$\begin{aligned} Ra &= 6.33 \times 10^{-1} \times (p/\mathrm{mmHg})^2 \\ &= 3.56 \times 10^{-5} \times (p/\mathrm{Pa})^2. \end{aligned} \tag{7}$$

Using Fig. 5, the dependence of temperature on pressure can be converted to that on Ra, and Figs. 6 and 7 can be obtained. Fig. 6 is the dependence of normalized temperature on Ra corresponding to Fig. 3. Fig. 7 is the dependence of normalized temperature distribution on Ra. This figure was obtained from Fig. 4 by the fifth-order polynomial fitting at each Ra using seven temperatures including both hot and cold wall temperatures. The order of this polynomial fitting is crucial to estimate heat flux. In the present study, this was decided to satisfy that the calculated temperatures at both walls are the nearest possible values as the constantly maintained values, or $\Theta = 1$ at $Z = 0$ and $\Theta = 0$ at $Z = 1$.

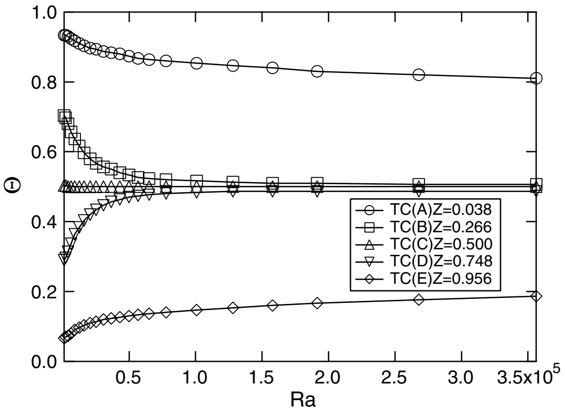

Fig. 6. Dependence of normalized temperature on *Ra*.

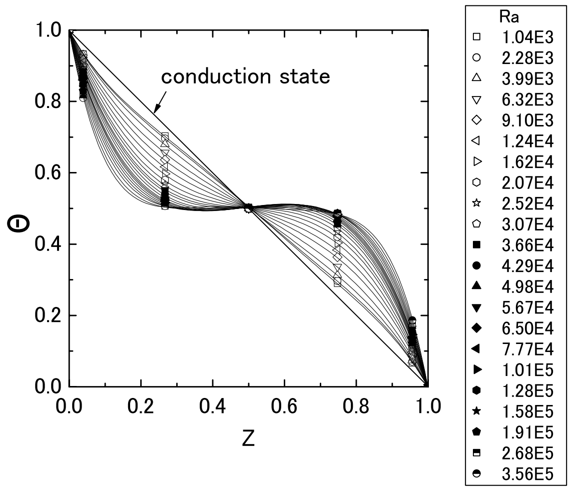

Fig. 7. Dependence of normalized temperature distribution on *Ra*. This figure is obtained from Fig. 4 by the fifth-order polynomial fitting at each *Ra* using seven temperatures including both wall temperatures. This was decided to satisfy that the calculated temperatures at both walls are the nearest possible values as the experimental conditions.

3.4. Data reduction by Nu

Though the temperature distribution at lower pressure approaches that at conduction state, this can be also illustrated from the viewpoint of heat flux using the Nusselt number (*Nu*). *Nu* is defined as the ratio of heat flux due to convection to that at conduction state. In accordance with Fig. 7, in the present study, *Nu* can be predicted by use of the following equation at the center of the hot and cold walls,

$$Nu = \frac{\left(\frac{\partial\theta}{\partial z}\right)_{\text{conv}}}{\left(\frac{\partial\theta}{\partial z}\right)_{\text{cond}}} = \frac{\left(\frac{\partial\Theta}{\partial Z}\right)_{\text{conv}}}{\left(\frac{\partial\Theta}{\partial Z}\right)_{\text{cond}}} = -\left(\frac{\partial\Theta}{\partial Z}\right)_{\text{conv}}. \tag{8}$$

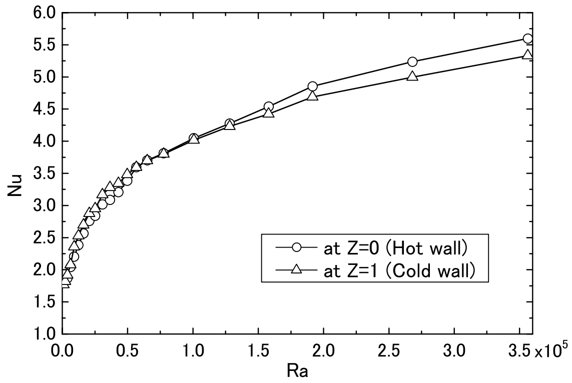

Fig. 8. Dependence of *Nu* on *Ra*. *Nu* is calculated at the center of the hot and cold walls using the polynomial fitted curves in Fig. 7.

Fig. 8 shows the dependence of *Nu* on *Ra*. *Nu* at the center of the hot and cold walls can be calculated from the derivatives of the temperatures using the polynomial fitted curves at $Z = 0$ for the hot wall, and $Z = 1$ for the cold wall. *Nu* at both walls are in good agreement at each *Ra* within 5%. This result suggests that the heat flux through the hot wall can be delivered to the cold wall almost without heat loss in the present experiment. From this figure, *Nu* appears to approach unity, or conduction state as *Ra* decreases.

4. Conclusion

Temperature distribution of air was measured for natural convection in a 50 mm cube imposed the temperature difference of 30 K between two opposing vertical walls at the average temperature of 298.15 K under the decreased pressure less than atmospheric one. *Ra* was changed by decreasing the pressure in the range where the air can be assumed continuous from the viewpoint of kinetic theory.

The experimentally obtained result illustrates well that the profiles of temperature distribution and heat flux approach those at conduction state as pressure or *Ra* becomes small. For the pressure from 5.40 kPa (40.5 mmHg) to 99.99 kPa (750.0 mmHg) which gives *Ra* from 1.04×10^3 to 3.56×10^5, *Nu* at the center of the hot and cold walls is almost less than 5.5 by the fifth-order polynomial fitted curves of temperature distribution using seven temperatures at experimentally obtained five measuring points and both walls.

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